Phenomenology of Planck-scale Lorentz-symmetry test theories

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In the recent quantum-gravity literature there has been strong interest in the possibility of Planck-scale departures from Lorentz symmetry, including possible modifications of the energy/momentum dispersion relation. I stress that a meaningful characterization of the progress of experimental bounds on these Planck-scale effects requires the analysis of some reference test theories, and I propose to focus on two "minimal" test theories, a pure-kinematics test theory and an effective-field-theory-based test theory. I illustrate some features of the phenomenology based on these test theories considering some popular strategies for constraining Planck-scale effects, and in particular I observe that sensitivities that are already in the Planck-scale range for some parameters of the two test theories can be achieved using observations of TeV photons from Blazars, both using the so-called "gamma-ray time-of-flight analyses" and using the now robust evidence of absorption of TeV photons. Instead the Crab-nebula synchrotron-radiation analyses, whose preliminary sensitivity estimates raised high hopes, actually do not lead to any bound on the parameters of the two "minimal" test theories. The Crab-nebula synchrotron-radiation analyses do however constrain some possible generalizations of one of the minimal test theories. As an example of forthcoming data which could provide extremely stringent (beyond-Planckian) limits on the two minimal test theories I consider the possibility of studies of the GZK cutoff for cosmic-rays.

I. DIFFERENT PERSPECTIVES ON THE FATE OF LORENTZ SYMMETRY IN QUANTUM SPACETIME

The fact that Lorentz symmetry is such a crucial ingredient of our present description of the fundamental laws of physics has motivated a large effort to test this symmetry to the highest possible precision. In addition to the general interest in probing the robustness of the principles we hold as fundamental, recently tests of Lorentz symmetry have attracted interest also as a result of the realization that in various approaches to the quantum-gravity problem one encounters nonclassical features of spacetime that lead to small departures from Lorentz symmetry. A quantum-gravity-motivated phenomenology of departures from Lorentz symmetry was proposed in Ref. [1]. The idea that Lorentz symmetry might be only an approximate symmetry has then been considered in quantum-gravity models based on spacetime foam pictures [2], in loop quantum gravity models [3,4] and in noncommutative geometry models [5–9], including some scenarios for noncommutative geometry that are relevant in string theory [6,7].

At a strictly phenomenological level one can view this interest in possible Planck-scale departures from Lorentz symmetry as originating from the idea that the sought quantum gravity might involve some sort of "granularity" of spacetime ("spacetime quanta"), and on the basis of experience with certain physical systems (especially condensed-matter systems) one can expect that granularity of the medium in which propagation occurs might lead to energy-dependent corrections [1] to the dispersion relation. At energies much larger than the particle mass but smaller than the granularity (Plankian) energy scale, the dispersion relation could be of the type[1]

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \left(\frac{E^n}{E_p^n} \right) + O(\frac{E^{n+3}}{E_p^{n+1}}) \tag{1}$$

where $E_p \simeq 1.2 \cdot 10^{16} TeV$ is the Planck scale, η parametrizes the ratio between the Planck scale and the scale of quantization of spacetime, and the power n is a key characteristic of the magnitude of the effects to be expected.

Of course, different intuitions for the right path toward quantum gravity may lead to different expectations with respect to this possible departures from Lorentz symmetry. And it is useful to realize that each quantum-gravity

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[1]In the literature the correction term is treated equivalently as a $\vec{p}^2 E^n$ correction and as a E^{n+2} correction, since one is anyway only interested in leading-order corrections in processes involving high-energy ($\vec{p}^2 \simeq E^2$) particles. Of course, the symbol m is meaningful as the rest energy of the particle (a low-energy concept) only if the correction term vanishes for particles at rest (the case of a $\vec{p}^2 E^n$ correction).

research line can be connected with one of three perspectives on the problem: the particle-physics perspective, the general-relativity perspective and the condensed-matter perspective.

From a particle-physics perspective it is natural to attempt to reproduce as much as possible the successes of the Standard Model of particle physics. One is tempted to see gravity simply as one more gauge interaction. From this particle-physics perspective a natural solution of the quantum gravity problem would be String-Theory-like: a quantum gravity whose core features are essentially described in terms of graviton-like exchange in a background classical spacetime. And from this particle-physics perspective there is clearly no in-principle reason to renounce to exact Lorentz symmetry, at least as long as Minkowski classical spacetime is an admissible background spacetime. Still, a breakup of Lorentz symmetry, in the sense of spontaneous symmetry breaking, is of course possible. And this possibility has been studied extensively [6,7] over the last few years, particularly in String Theory, which is the most mature quantum-gravity approach that emerged from the particle-physics perspective.

The general-relativity perspective naturally leads to reject the use of a background spacetime, and this is widely acknowledged [10–13]. Although less publicized, there is also growing awareness of the fact that the development of general relativity relied heavily on the careful consideration of the in-principle limitations that measurement procedures can encounter. Think for example of the limitations that the speed-of-light limit imposes on certain setups for clock synchronization. In light of the various arguments suggesting that, whenever both quantum mechanics and general relativity are taken into account, there should be an in-principle limitation to the localization of a spacetime point (an event), the general-relativity perspective invites one to renounce to any direct reference to a classical spacetime [14–17]. Indeed this requirement that the in-principle measurability limitations be reflected by the adoption of a corresponding measurability-limited description of spacetime, is another element of intuition which is guiding quantum-gravity research from the general-relativity perspective. This naturally leads one to consider certain types of discretized spacetimes, as in the Loop Quantum Gravity approach [10–13], or noncommutative spacetimes [14,15]. Results obtained over the last few years indicate that from this general-relativity perspective some Planck-scale departures from Lorentz symmetry are naturally expected (although not automatic). Loop quantum gravity and other discretized-spacetime quantum-gravity approaches appear to require [3,4,20] some departures, governed by the Planck scale, from the familiar (continuous) Lorentz symmetry. And Planck-scale departures from Lorentz symmetry might be inevitable in noncommutative spacetimes, as shown in several recent studies [6–9,18,19].

The third possibility is a condensed-matter perspective (see, *e.g.*, the research programs of Refs. [21] and [22]) on the quantum-gravity problem, in which some of the familiar properties of spacetime are only emergent. Condensed-matter theorists are used to describe some of the degrees of freedom that are measured in the laboratory as collective excitations within a theoretical framework whose primary description is given in terms of much different, and often practically unaccessible, fundamental degrees of freedom. Close to a critical point some symmetries arise for the collective-excitations theory, but these symmetries do not carry the significance of fundamental symmetries, and are in fact lost as soon as the theory is probed somewhat away from the critical point. Notably, some familiar systems are known to exhibit special-relativistic invariance in certain limits, even though, at a more fundamental level, they are described in terms of a nonrelativistic theory. Clearly from this (relatively new) condensed-matter perspective on the quantum-gravity problem it is natural to see the familiar classical continuous Lorentz symmetry only as an approximate (emergent) symmetry.

The interest in testing the idea of Planck-scale departures from Lorentz symmetry is also due to the differences between these alternative perspectives on the quantum-gravity problem. Any experimental hint on the fate of Lorentz symmetry at the Planck scale might allow us to establish which (if any) of these perspectives on the quantum-gravity problem is to be favoured.

II. PLANCK-SCALE DEPARTURES FROM LORENTZ SYMMETRY, MODIFIED DISPERSION RELATIONS AND TEST THEORIES

Motivation for the study of Planck-scale departures from Lorentz symmetry does not simply come from the mentioned general perspectives on the quantum-gravity problem: for some of the theoretical frameworks that are being considered in quantum-gravity research evidence of such departures from Lorentz symmetry has been found. In this section I start with a brief description of how modified dispersion relations arise[2] in the study of noncommutative

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[2]I discuss noncommutative spacetimes and the Loop Quantum Gravity approach, which are the best understood Planck-scale frameworks in which it appears that the dispersion relation is Planck-scale modified. But other types of intuitions about the quantum-gravity problem may lead to modified dispersion relations, including some realizations of the idea of "spacetime

spacetimes and in the study of loop quantum gravity.

These results should provide guidance in setting up a phenomenology for the Planck-scale departures from Lorentz symmetry. In particular I want to stress that it is necessary for this phenomenology to rely on some reference test theories, which should be inspired by the results obtained in the study of noncommutative spacetimes and in the study of loop quantum gravity. In the second part of this section I discuss two such test theories on which it might be appropriate to focus.

A. Modified dispersion relations in canonical noncommutative spacetime

The noncommutative spacetimes in which modifications of the dispersion relation are being most actively considered all fall within the following rather general parametrization of noncommutativity of the spacetime coordinates:

$$[x_\mu, x_\nu] = i\theta_{\mu\nu} + i\rho_{\mu\nu}^\beta x_\beta \ . \tag{2}$$

It is convenient to first focus on the special case $\rho = 0$, the "canonical noncommutative spacetimes"

$$[x_\mu, x_\nu] = i\theta_{\mu\nu} \ . \tag{3}$$

Of course, the natural first guess for introducing dynamics in these spacetimes is a quantum field theory formalism. And indeed, for the special case $\rho = 0$, an approach to the construction of a quantum field theory has been developed rather extensively [6,7]. While most aspects of these field theories closely resemble their commutative-spacetime counterparts, a surprising feature that emerges is the so-called "IR/UV mixing" [6,7,25]: the high-energy sector of the theory does not decouple from the low-energy sector. Connected with this IR/UV mixing is the type of modified dispersion relations that one encounters in field theory on canonical noncommutative spacetime, which in general take the form

$$m^2 \simeq E^2 - \vec{p}^2 + \frac{\alpha_1}{p^\mu \theta_{\mu\nu} \theta^{\nu\sigma} p_\sigma} + \alpha_2 m^2 \ln\left(p^\mu \theta_{\mu\nu} \theta^{\nu\sigma} p_\sigma\right) + \dots \ , \tag{4}$$

where the α_i are parameters, possibly taking different values for different particles (the dispersion relation is not "universal"), that depend on various aspects of the field theory, including its field content and the nature of its interactions. The fact that this dispersion relation can be singular in the infrared is a result of the IR/UV mixing. A part of the infrared singularity could be removed by introducing (exact) supersymmetry, which typically leads to $\alpha_1 = 0$.

The implications of this IR/UV mixing for dynamics are still not fully understood, and there is still justifiable skepticism [26] toward the reliability of the type of field-theory construction adopted so far. I think it is legitimate to even wonder whether a field-theoretic formulation of the dynamics is at all truly compatible with the canonical spacetime noncommutativity. The Wilson decoupling between IR and UV degrees of freedom is a crucial ingredient of most applications of field theory in physics, and it is probably incompatible with canonical noncommutativity: the associated uncertainty principle of the type $\Delta x_\mu \Delta x_\nu \geq \theta_{\mu\nu}$ implies that it is not possible to probe short distances (small, say, Δx_1) without probing simultaneously the large-distance regime ($\Delta x_2 \geq \theta_{2,1}/\Delta x_1$).

In any case, the presence of modified dispersion relations in canonical noncommutative spacetime should be expected, since Lorentz symmetry is "broken" by the tensor $\theta_{\mu\nu}$. An intuitive characterization of this Lorentz-symmetry breaking can be obtained by looking at wave exponentials. The Fourier theory in canonical noncommutative spacetime is based [27] on simple wave exponentials $e^{ip^\mu x_\mu}$ and from the $[x_\mu, x_\nu] = i\theta_{\mu\nu}$ noncommutativity relations one finds that

$$e^{ip^\mu x_\mu} e^{ik^\nu x_\nu} = e^{-\frac{i}{2} p^\mu \theta_{\mu\nu} k^\nu} e^{i(p+k)^\mu x_\mu} \ , \tag{5}$$

i.e. the Fourier parameters p_μ and k_μ combine just as usual, but there is the new ingredient of the overall θ-dependent phase factor.

The fact that momenta combine in the usual way reflects the fact that the transformation rules for energy-momentum from one (inertial) observer to another are still the familiar, undeformed, Lorentz transformation rules. However, the product of wave exponentials depends on $p^\mu \theta_{\mu\nu} k^\nu$; it depends on the "orientation" of the energy-momentum vectors

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foam" [1,2,23], which allow an analogy with the laws of particle propagation in a thermal environment [1,2,24].

p^μ and k^ν with respect to the $\theta_{\mu\nu}$ tensor. The $\theta_{\mu\nu}$ tensor plays the role of a background that identifies a preferred class of inertial observers[3]. Different particles can be affected by the presence of this background in different ways, leading to the emergence of different dispersion relations. All this is consistent with indications of the mentioned popular field theories in canonical noncommutative spacetimes.

B. Modified dispersion relations in κ-Minkowski noncommutative spacetime

In canonical noncommutative spacetimes Lorentz symmetry is "broken" and there is growing evidence that Lorentz symmetry breaking occurs for most choices of the tensors θ and ρ. It is at this point clear, in light of several recent results, that the only way to preserve Lorentz symmetry is the choice $\theta = 0 = \rho$, *i.e.* the case in which there is no noncommutativity and one is back to the familiar classical commutative Minkowski spacetime. When noncommutativity is present Lorentz symmetry is usually broken, but recent results suggest that for some special choices of the tensors θ and ρ Lorentz symmetry might be deformed, in the sense of the recently proposed "doubly-special relativity" scenario [8], rather than broken. In particular, this appears to be the case for the Lie-algebra κ-Minkowski [18,19,5,28–30] noncommutative spacetime $(l, m = 1, 2, 3)$

$$[x_m, t] = \frac{i}{\kappa} x_m \ , \qquad [x_m, x_l] = 0 \ . \tag{6}$$

κ-Minkowski is a Lie-algebra spacetime that clearly enjoys classical space-rotation symmetry; moreover, at least in a Hopf-algebra sense (see, *e.g.*, Ref. [29]), κ-Minkowski is invariant under "noncommutative translations". Since I am focusing here on Lorentz symmetry, it is particularly noteworthy that in κ-Minkowski boost transformations are necessarily modified [29]. A first hint of this comes from the necessity of a deformed law of composition of momenta, encoded in the so-called coproduct (a standard structure for a Hopf algebra). One can see this clearly by considering the Fourier tranform. It turns out [5,28] that in the κ-Minkowski case the correct formulation of the Fourier theory requires a suitable ordering prescription for wave exponentials. From

$$: e^{ik^\mu x_\mu} :\equiv e^{ik^m x_m} e^{ik^0 x_0} \ , \tag{7}$$

as a result of $[x_m, t] = ix_m/\kappa$ (and $[x_m, x_l] = 0$), it follows that the wave exponentials combine in a nontrivial way:

$$(: e^{ip^\mu x_\mu} :)(: e^{ik^\nu x_\nu} :) =: e^{i(p\dot{+}k)^\mu x_\mu} : \quad . \tag{8}$$

The notation "$\dot{+}$" here introduced reflects the behaviour of the mentioned "coproduct" composition of momenta:

$$p_\mu \dot{+} k_\mu \equiv \delta_{\mu,0}(p_0 + k_0) + (1 - \delta_{\mu,0})(p_\mu + e^{\lambda p_0} k_\mu) \ . \tag{9}$$

As argued in Refs. [8] the nonlinearity of the law of composition of momenta might require an absolute (observer-independent) momentum scale, just like upon introducing a nonlinear law of composition of velocities one must introduce the absolute observer-independent scale of velocity c. The inverse of the noncommutativity scale λ should play the role of this absolute momentum scale. This invites one to consider the possibility [8] that the transformation laws for energy-momentum between different observers would have two invariants, c and λ, as required in "doubly-special relativity" [8].

On the basis of (9) one is led [18,19,5] to the following result for the form of the energy/momentum dispersion relation

$$\left(\frac{2}{\lambda} \sinh \frac{\lambda m}{2}\right)^2 = \left(\frac{2}{\lambda} \sinh \frac{\lambda E}{2}\right)^2 - e^{\lambda E} \vec{p}^2 \ , \tag{10}$$

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[3]Note that these remarks apply to canonical noncommutative spacetimes as studied in the most recent (often String-Theory inspired) literature, in which $\theta_{\mu\nu}$ is indeed simply a tensor (for a given observer, an antisymmetric matrix of numbers). I should stress however that the earliest studies of canonical noncommutative spacetimes (see Ref. [14] and follow-up work) considered a $\theta_{\mu\nu}$ with richer mathematical properties, notably with nontrivial algebra relations with the spacetime coordinates. In that earlier, and more ambitious, setup it is not obvious that Lorentz symmetry would be broken: the fate of Lorentz symmetry may depend on the properties attributed to $\theta_{\mu\nu}$.

which for low momenta takes the approximate form

$$m^2 \simeq E^2 - \vec{p}^2 - \lambda E \vec{p}^2 \ . \tag{11}$$

Actually, the precise form of the dispersion relation may depend on the choice of ordering prescription for wave exponentials [29] ((10) follows form (7)), and this point deserves further scrutiny. But even setting aside this annoying ordering ambiguity, there appear to be severe obstructions [28,29] for a satisfactory formulation of a quantum field theory in κ-Minkowski. The techniques that were rather straightforwardly applied for the construction of field theory in canonical noncommutative spacetime do not appear [28,29] to be applicable in the κ-Minkowski case. It is not unplausible that the "virulent" κ-Minkowski noncommutativity may require some departures from a standard field-theoretic setup.

C. Modified dispersion relation in Loop Quantum Gravity

Loop Quantum Gravity is one of the most ambitious approaches to the quantum-gravity problem, and its understanding is still in a relatively early stage. As presently understood, Loop Quantum Gravity predicts an inherently discretized spacetime [10–12], and this occurs in a rather compelling way: it is not that one introduces by hand an *a priori* discrete background spacetime; it is rather a case in which a background-independent analysis ultimately leads, by a sort of self-consistency, to the emergence of discretization. There has been much discussion recently, prompted by the studies [1,3,4], of the possibility that this discretization might lead to broken Lorentz symmetry and a modified dispersion relation. Although there are cases in which a discretization is compatible with the presence of continuous classical symmetries [31,32], it is of course natural, when adopting a discretized spacetime, to put Lorentz symmetry under careful scrutiny. Arguments presented in Refs. [3,4] suggest that Lorentz symmetry might indeed be broken in Loop Quantum Gravity.

Moreover, very recently Smolin, Starodubtsev and I proposed [20] (also see the follow-up study in Ref. [33]) a mechanism such that Loop Quantum Gravity would be described at the most fundamental level as a theory that in the flat-spacetime limit admits deformed Lorentz symmetry, in the sense of the "doubly-special relativity" scenario [8]. Our argument originates from the role that certain quantum symmetry groups ("q-deformed algebras") have in the Loop-Quantum-Gravity description of spacetime with a cosmological constant, and observing that in the flat-spacetime limit (the limit of vanishing cosmological constant) these quantum groups might not contract to a classical Lie algebra, but rather contract to a quantum (Hopf) algebra.

All these studies point to the presence of a modified dispersion relation, although different arguments lead to different intuition for the form of the dispersion relation. A definite result might have to wait for the solution of the well-known "classical-limit problem" of Loop Quantum Gravity. We are presently unable to recover from this full quantum-gravity theory the limiting case in which the familiar quantum-field-theory description of particle-physics processes in a classical background spacetime applies. Some recent results [34,35], which tackle the problem of reproducing Fock-space quantization from the Loop-Quantum-Gravity framework, may provide the first ingredients of such a formulation. But other recent studies appear to suggest [36] that in the same contexts in which departures from Lorentz symmetry may be revealed one should adopt a density-matrix formalism, and then the whole picture would collapse to the familiar Lorentz-invariant quantum-field-theory description in contexts involving both relatively low energies and relatively low boosts with respect to the center-of-mass frame (e.g. the particle-physics collisions studied at several particle accelerators).

D. Some issues relevant for the proposal of test theories

While the first few years of work on this idea of Planck-scale departures from Lorentz symmetry were necessarily based on rather preliminary analyses, with the only objective of establishing the point that Planck-scale sensitivity could be achieved in some cases, I want to stress that we should now gear up for a more "mature" phase of work on quantum-gravity phenomenology, in which the development and analysis of some carefully crafted test theories takes center stage. The results I briefly summarized in the previous three subsections suggest that in the analysis of noncommutative spacetimes and in the analysis of Loop Quantum Gravity, the two approaches that provide most of the motivation for this phenomenology, we are getting closer to obtaining truly characteristic predictions, prediction that could be used to falsify the corresponding theoretical scheme. But there are a few open issues which do not at present allow us to describe in detail a falsifiable prediction, and therefore, for now, the phenomenology must rely on some appropriately structured test theories. These test theories should on the one hand reflect the points we do

understand of these quantum-gravity approaches and on the other hand they should limit as much as possible the risk of assuming properties that could turn out not to be verified once we understand the formalisms better.

The test theories should also be used for bridging the gap between the experimental data and the analysis of the formalisms. The test theories should provide a common language in assessing the progresses made in improving the sensitivity of experiments, a language that must also be suitable for access from the side of those working at the development of the quantum-gravity/quantum-spacetime theories.

As we contemplate the challenge of developing such carefully-balanced test theories it is important to observe that the most robust part of the results I summarized in the previous three subsections is clearly the emergence of modified dispersion relations. Therefore if one could set up experiments testing directly the dispersion relation the resulting limits would have wide applicability. In principle one could investigate the form of the dispersion relation directly by making simultaneous measurements of energy and space-momentum; however, it is easy to see that achieving Planck-scale sensitivity in such a direct test is well beyond our capabilities.

Useful test theories on which to base the relevant phenomenology must therefore combine the ingredient of the dispersion relation with other ingredients. As I shall discuss in greater detail later in this section, there are three key issues for this test-theory development:

(i) in presence of the modified dispersion relation should we still assume the validity of the relation $v = dE/dp$ between the speed of a particle and its dispersion relation? (here dE/dp is the derivative of the function $E(p)$ which of course is implicitly introduced through the dispersion relation)

(ii) in presence of the modified dispersion relation should we still assume the validity of the standard law of energy-momentum conservation?

(iii) in presence of the modified dispersion relation which formalism should be adopted for the description of dynamics?

The fact that these are key issues is also a consequence of the type of data that we expect to have access to, as I shall discuss here later.

Unfortunately on these three key points the quantum-spacetime pictures which are providing motivation for the study of Planck-scale modifications of the dispersion relation, reviewed in the previous three subsections, are not providing much guidance yet.

For example, in Loop Quantum Gravity, while we do have evidence that the dispersion relation should be modified, we do not yet have a clear indication concerning whether the law of energy-momentum conservation should also be modified and we also cannot yet robustly establish whether the relation $v = dE/dp$ should be preserved. Moreover, the "classical-limit problem", as mentioned, also affects the choice of formalism to be adopted for the description of dynamics. It is not at all clear how and in which regimes a field-theoretic setup should be available, and some recent studies appear to suggest [36] that in the same contexts in which departures from Lorentz symmetry may be revealed one should also adopt a density-matrix formalism. We should therefore be prepared for surprises in the description of dynamics.

Similarly in the analysis of noncommutative spacetimes we are close to establishing in rather general terms that some modification of the dispersion relation is inevitable, but other aspects of the framework have not yet been clarified. While most of the literature for canonical noncommutative spacetimes assumes [6,7] that the law of energy-momentum conservation should not be modified, most of the literature for κ-Minkowski spacetime argues in favour of a modification (perhaps consistent with the corresponding doubly-special-relativity criteria [8]) of the law of energy-momentum conservation. There is also still no consensus on the relation between speed and dispersion relation, and particularly in the κ-Minkowski literature some departures from the $v = dE/dp$ relation are actively considered [37–40]. And concerning the formalism to be used for the description of dynamics in a noncommutative spacetime, while everybody's first guess is the field-theoretic formalism, the fact that attempts at a field theory formulation encounter so many difficulties (the IR/UV mixing for the canonical-noncommutative spacetime case and the even more pervasive shortcomings of the proposals for a field theory in κ-Minkowski) must invite one to contemplate possible alternative formulations of dynamics.

Clearly the situation on the theory side invites us to be prudent: if a given phenomenological picture relies on too many assumptions on Planck-scale physics it is likely that it might not reproduce any of the mentioned quantum-gravity and/or quantum-spacetime models (when these models are eventually fully understood they will give us their own mix of Planck-scale features, which is difficult to guess at the present time). On the other hand it is necessary for the robust development of a phenomenology to adopt well-defined test theories. Without reference to a well-balanced set of test theories it is impossible to compare the limits obtained in different experimental contexts, since each experimental context may require different "ingredients" of Planck-scale physics. And it is of course meaningless to compare limits obtained on the basis of different conjectures for the Planck-scale regime.

E. A test theory for pure kinematics

The majority (see, *e.g.*, Refs. [42–46]) of studies concerning Planck-scale modifications of the dispersion relation adopt the phenomenological formula

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \left(\frac{E^n}{E_p^n} \right) + O(\frac{E^{n+3}}{E_p^{n+1}}) \ , \tag{12}$$

with real η of order 1 and integer n. This formula is compatible with some of the results obtained in the Loop-Quantum-Gravity approach and reflects the results obtained in κ-Minkowski and other noncommutative spacetimes (but, as mentioned, in the special case of canonical noncommutative spacetimes one encounters a different, infrared singular, dispersion relation).

As mentioned, on the basis of the status on the theory side, a prudent approach in combining the dispersion relation with other ingredients is to be favoured. Since basically all experimental situations will involve some aspects of kinematics that go beyond the dispersion relation (while there are some cases in which the dynamics, the interactions among particles, does not play a role), and taking into account the mentioned difficulties in establishing what is the correct formalism for the description of dynamics[4] at the Planck scale, most authors prefer to prudently combine the dispersion relation with other "purely kinematical" aspects of Planck-scale physics.

Already in the first studies [1] that proposed a phenomenology based on (12) it was assumed that the dispersion relation would still be "universal" (same for all particles) and that even at the Planck scale the familiar description of "group velocity", obtained from the dispersion relation according to $v = dE/dp$, should hold[5].

In other works motivated by the analysis reported in Ref. [1] another key kinematical feature was introduced: starting with the studies reported in Refs. [43–46] the dispersion relation (12) and the velocity relation $v = dE/dp$ were combined with the assumption that the law of energy-momentum conservation should not be modified at the Planck scale, so that, for example, in a $a + b \rightarrow c + d$ particle-physics process one would have

$$E_a + E_b = E_c + E_d \ , \tag{13}$$

$$\vec{p}_a + \vec{p}_b = \vec{p}_c + \vec{p}_d \ . \tag{14}$$

The elements I described in this Section compose a quantum-gravity phenomenology test theory that has already been widely considered in the literature, even though it was never previously characterized in detail. In the following I will refer to this test theory as the "minimal AEMNS test theory"[6], and I will assume that experimental bounds on this test theory should be placed by using only the following assumptions:

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[4]I am here using the expression "dynamics at the Planck scale" with some license. Of course, in our phenomenology we will not be sensitive directly to the dynamics at the Planck scale. However, as I discuss in greater detail in the next subsection, if the arguments that encourage the use of new descriptions of dynamics at the Planck scale are correct, then a sort of "order of limits problem" clearly arises. Our experiments will involve energies much lower than the Planck scale, and we know that in the infrared limit the familiar formalism with field-theoretic description of dynamics and Lorentz invariance will hold. So we would need to establish whether experiments that are sensitive to Planck-scale departures from Lorentz symmetry could also be sensitive to Planck-scale departures from the field-theoretic description of dynamics. Since we still know very little about this alternative descriptions of dynamics a prudent approach, avoiding any assumption about the description of dynamics is certainly preferable.

[5]As mentioned, this assumption is not guaranteed to apply to the formalisms of interest, and indeed several authors have considered alternatives [37–40]. While the studies advocating alternatives to $v = dE/dp$ rely of a large variety of arguments (some more justifiable some less), in my own perception [47] a key issue here is whether quantum gravity leads to a modified Heisenberg uncertainty principle, $[x, p] = 1 + F(p)$. Assuming a Hamiltonian description is still available, $v = dx/dt \sim [x, H(p)]$, the relation $v = dE/dp$ essentially follows from $[x, p] = 1$. But if $[x, p] \neq 1$ then $v = dx/dt \sim [x, H(p)]$ would not lead to $v = dE/dp$. And there is much discussion in the quantum-gravity community of the possibility of modifications of the Heisenberg uncertainty principle at the Planck scale.

[6]I am using "AEMNS" on the basis of the initials of the names of the authors in Ref. [1], which first proposed a phenomenology based on the dispersion relation (12). But as mentioned the full test theory, as presently used in most studies, only emerged gradually in follow-up work. In particular, there was no discussion of energy-momentum conservation in Ref. [1]. Unmodified energy-momentum conservation was introduced in Refs. [43–46]. Concerning this "minimal AEMNS test theory" it should also be noticed that Ellis, Mavromatos and Nanopoulos actually favour [48] a quantum-gravity approach in which the modification of the dispersion relation is not universal, and therefore would not fit within the confines of the "minimal AEMNS test theory" (although, of course, nonuniversality can be accommodated in a straightforward generalization of the test theory).

(minAEMNS.1) the dispersion relation is of the form

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 \left(\frac{E^n}{E_p^n} \right) + O(\frac{E^{n+3}}{E_p^{n+1}}) \, , \qquad (15)$$

where η and n are universal (same value for every particle and for both helicities/polarizations of a given particle);

(minAEMNS.2) the velocity of a particle can be obtained from the dispersion relation using $v = dE/dp$;

(minAEMNS.3) the law of energy-momentum conservation is not modified;

(minAEMNS.4) nothing is assumed about dynamics (*i.e.* the analysis of this test theory will be limited to classes of experimental data that involve pure kinematics, without any role for dynamics).

While this "minimal" version of the test theory appears to deserve to be the primary focus of AEMNS-based phenomenology work, it is of course legitimate to consider some possible generalizations, including a nonuniversality of the effects (allowing for different values of the dispersion-relation-modification parameters for different particles).

F. A test theory based on low-energy effective field theory

The AEMNS test theory has the merit of relying only on a relatively small network of assumptions on kinematics, without assuming anything about the role of the Planck scale in dynamics. However, of course, this justifiable prudence turns into a severe limitation on the class of experimental contexts which can be used to constrain the parameters of the test theory. It is in fact rather rare that a phenomenological analysis can be completed without using any aspects of the interactions among the particles involved in the relevant processes. The desire to be able to analyze a wider class of experimental contexts is therefore providing motivation for the development of test theories more ambitious than the AEMNS test theory, with at least some elements of dynamics. This is understandable but, in light of the situation on the theory side, work with one of these more ambitious test theories should proceed with the awareness that there is a high risk that it may turn out that none of the quantum-gravity approaches which are being pursued is reflected in the test theory.

One reasonable possibility to consider, when the urge to analyze data that involve some contamination from dynamics cannot be resisted, is the one of describing dynamics within the framework of low-energy effective field theory. In this subsection I want to discuss a test theory which is indeed based on low-energy effective field theory, and has emerged from the work recently reported in Ref. [49] (which is rooted in part in the earlier Ref. [3]).

Before a full characterization of this test theory I should first warn the reader that there might be some severe limitations for the applicability of low-energy effective field theory to the investigation of Planck-scale physics, especially when departures from Lorentz symmetry are present.

A significant portion of the quantum-gravity community is in general, justifiably, skeptical about the results obtained using low-energy effective field theory in analyses relevant for the quantum-gravity problem. After all the first natural prediction of low-energy effective field theory in the gravitational realm is a value of the energy density which is some 120 orders of magnitude greater than allowed by observations[7].

As a result of the different perspectives on the quantum-gravity problem, which I already described in Section 1, there are on the one hand numerous researchers who are skeptical about any results obtained using low-energy effective field theory in analyses relevant for the quantum-gravity problem, but there are on the other hand also quite a few researchers interested in the quantum-gravity problem who are completely serene in assuming that all quantum-gravity effects should be describable in terms of effective field theory in low-energy situations.

I feel that, while of course an effective-field-theory description may well turn out to be correct in the end, the *a priori* assumption that a description in terms of effective low-energy field-theory should work is rather naive. If the arguments that encourage the use of new descriptions of dynamics at the Planck scale are correct, then a sort of "order of limits problem" clearly arises. Our experiments will involve energies much lower than the Planck scale, and we know that in some limit (a limit that characterizes our most familiar observations) the field-theoretic description and Lorentz invariance will hold. So we would need to establish whether experiments that are sensitive to Planck-scale departures from Lorentz symmetry could also be sensitive to Planck-scale departures from the field-theoretic

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[7]And the outlook of low-energy effective field theory in the gravitational realm does not improve much through the observation that exact supersymmetry could protect from the emergence of any energy density. In fact, Nature clearly does not have supersymmetry at least up to the TeV scale, and this would still lead to a natural prediction of the cosmological constant which is some 60 orders of magnitude too high.

description of dynamics. As an example, let me mention the possibility (not unlikely in a context which is questioning the fate of Lorentz symmetry) that quantum gravity would admit a field-theory-type description only in reference frames in which the process of interest is essentially occurring in its center of mass (no "Planck-large boost" [52] with respect to center-of-mass frame). The field theoretic description could emerge in a sort of "low-boost limit", rather than the expected low-energy limit. The regime of low boosts with respect the center-of-mass frame is often indistinguishable with respect to the low-energy limit. For example, from a Planck-scale perspective, our laboratory experiments (even the ones conducted at, e.g. CERN, DESY, SLAC...) are both low-boost (with respect to the center of mass frame) and low-energy. However, the "UHE cosmic-ray paradox", for which a quantum-gravity origin has been conjectured (see later), occurs in a situation where all the energies of the particles are still tiny with respect to the Planck energy scale, but the boost with respect to the center-of-mass frame (as measured by the ratio E/m_{proton} between the proton energy and the proton mass) could be considered to be "large" from a Planck-scale perspective $(E/m_{proton} \gg E_p/E)$.

These concerns are strengthened by looking at the literature available on the quantum pictures of spacetime that provide motivation for the study of modified dispersion relations, which usually involve either noncommutative geometry or Loop Quantum Gravity, where, as mentioned, the outlook of a low-energy effective-field-theory description is not reassuring.

Of course, in phenomenology this type of concerns can be set aside, since one is primarily looking for confrontation with experimental data, rather than theoretical prejudice. It is clearly legitimate to set up a test theory exploring the possibility of Planck-scale departures from Lorentz symmetry within the formalism of low-energy effective field theory. But one must then keep in mind that the implications for most quantum-gravity research lines of the experimental bounds obtained in this way might be very limited. This will indeed be the case if we discover that, as some mentioned preliminary results suggest, the limit in which the full quantum-gravity theory reproduces a description in terms of effective field theory in classical spacetime is also the limit in which the departures from Lorentz symmetry must be neglected.

Having provided this long warning, let me now proceed to a characterization of the test theory which I see emerging from the works reported in Refs. [49,3]. These studies explore the possibility of a linear-in-L_p modification of the dispersion relation

$$m^2 \simeq E^2 - \vec{p}^2 + \eta \vec{p}^2 L_p E , \qquad (16)$$

i.e. the case $n = 1$ of Eq. (12). The key assumption in Refs. [49,3] is that such modifications of the dispersion relation should be introduced consistently with an effective low-energy field-theory description of dynamics. The implications of this assumption were explored in particular for fermions and photons[8]. It became quickly clear that in such a setup universality cannot be assumed, since one must at least accommodate a polarization dependence for photons: in the field-theoretic setup it turns out that when right-circular polarized photons satisfy the dispersion relation $E^2 \simeq p^2 + \eta_\gamma p^3$ then necessarily left-circular polarized photons satisfy the "opposite sign" dispersion relation $E^2 \simeq p^2 - \eta_\gamma p^3$. For spin-1/2 particles the analysis reported in Ref. [49] does not necessarily suggest a similar helicity dependence, but of course in a context in which photons experience such a complete correlation of the sign of the effect with polarization it would be awkward to assume that instead for electrons the effect is completely helicity independent. One therefore introduces two independent parameters η_+ and η_- to characterize the modification of the dispersion relation for electrons.

These observations provide the basis for a "GPMP test theory"[9]. However, as in the case of the minimal AEMNS test theory, it appears wise to first focus the phenomenology on a reduced two-parameter version of the test theory, reflecting some natural physical assumptions. As usual, once the reduced version of the test theory is falsified one can contemplate its possible generalizations.

In introducing a reduced GPMP test theory I believe that a key point of naturalness comes from the observation that the effective-field-theory setup imposes for photons a modification of the dispersion relation which has the same magnitude for both polarizations but opposite sign: it is then natural to give priority to the hypothesis that for

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[8]Actually the studies in Ref. [49,3] focus primarily on electrons and photons. I will assume that the results for photons generalize to all fermions, but I must warn the reader that the theory work needed to fully justify this (however natural) generalization is still in progress.

[9]Here, for the "GPMP" short name, I am guided by the initials of the authors of Refs. [49,3], but again those authors should not be held "responsible" for the entire structure of this GPMP framework I am describing. Actually, the studies reported in Refs. [49] and [3] differ significantly even among them: in particular, while Ref. [49] assumes locality of the new terms in the Lagrangian density, Ref. [3] also contemplates the possibility of nonlocality.

fermions a similar mechanism would apply, *i.e.* the modification of the dispersion relation should have the same magnitude for both signs of the helicity, but have a correlation between the sign of the helicity and the sign of the dispersion-relation modification. This would correspond to the natural-looking assumption that the Planck-scale effects are such that in a beam composed of randomly selected particles the average speed in the beam is still governed by ordinary special relativity (the Planck-scale effects average out summing over polarization/helicity).

A further "natural" reduction[10] of the parameter space is achieved by assuming that all fermions are affected by the same modification of the dispersion relation.

In the following I refer to this reduced two-parameter GPMP test theory as the "minimal GPMP test theory", characterized by the following ingredients:

(minGPMP.1) right-circular polarized photons are governed by the dispersion relation

$$m^2 \simeq E^2 - \vec{p}^2 + \eta_\gamma \vec{p}^2 \left(\frac{E}{E_p} \right) \ , \tag{17}$$

while left-circular polarized photons are governed by the dispersion relation

$$m^2 \simeq E^2 - \vec{p}^2 - \eta_\gamma \vec{p}^2 \left(\frac{E}{E_p} \right) \ ; \tag{18}$$

(minGPMP.2) for fermions the dispersion relation takes the form

$$m^2 \simeq E^2 - \vec{p}^2 + \eta_f \vec{p}^2 \left(\frac{E}{E_p} \right) \ , \tag{19}$$

in the positive-helicity case, while for negative-helicity fermions

$$m^2 \simeq E^2 - \vec{p}^2 - \eta_f \vec{p}^2 \left(\frac{E}{E_p} \right) \ , \tag{20}$$

with the same value of η_f for all fermions;

(minGPMP.3) dynamics is described in terms of effective low-energy field theory.

While this "minimal" version of the test theory may deserve to be the primary focus of GPMP-based phenomenology work, it is of course legitimate to consider its generalizations with independent parameters (rather than a single parameter, with the opposite-sign correlation) for the two helicities of fermions, and possibly allowing for different values of the parameters for different species of fermions.

III. SOME KEY ISSUES FOR PHENOMENOLOGY WITH THE TWO TEST THEORIES

I have argued that it is necessary to enter a new more mature phase of Quantum Gravity Phenomenology, in which the use of some reference test theories takes center stage. But I have also observed that there are a number of delicate issues that need to be considered in setting up such test theories. In the end, as a way to handle the large variety of scenarios we should be prepared to face as the understanding of quantum-gravity pictures progresses, it appeared wise to introduce both a pure-kinematics test theory and a field-theory-based test theory. But, while conceptually these two types of test theories are well motivated and well defined, one must face additional difficulties in developing a phenomenology based on these test theories. For the pure-kinematics test theory the difficulties originate primarily from the fact that sometimes an effect due to modification of dynamics can take a form that is not easily distinguished from a pure-kinematics effect. For the field-theory-based test theory the difficulties originate from the fact that the relevant field theory is not renormalizable.

In this section I want to stress these difficulties, but I also intend to argue that there is a reasonable way to proceed in spite of these difficulties.

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[10]I must here stress that, while it is "natural" to start the phenomenology from the assumption that all fermions experience the same Planck-scale effects, there are some "natural" mechanisms that could lead to a different magnitude of the effect for different types of fermions. As mentioned, one finds an example of such a possibility in the context of the analysis of certain approaches to field theory in canonical noncommutative spacetimes.

A. On the field-theory-based phenomenology

In introducing the "minimal GPMP test theory" in the previous section I stressed that the assumption of a description of dynamics in terms of effective field theory might fail to capture the insight gained from preliminary analysis of certain quantum-spacetime pictures. This "conceptual concern" can be easily set aside in phenomenology work. However, there is another, potentially more troublesome, issue that affects phenomenology work with the GPMP test theory: the relevant field theory is not renormalizable, and therefore, at least at a strict in-principle level, it is not predictive.

A description of possible Planck-scale departures from Lorentz symmetry within effective field theory can only be developed with a rather strongly pragmatic attitude; in fact, while one can introduce Planck-scale suppressed effects at tree level, one expects that loop corrections would typically lead, for fixed bare parameters and cutoff scale, to inadmissibly large departures from ordinary Lorentz symmetry. The parameters of the theory can be fine-tuned to eliminate the unwanted large effects, but the needed level of fine tuning is usually rather unpleasant. A particularly unpleasant level of fine tuning might be required in the case of the GPMP test theory since some authors (notably Refs. [50,51]) have argued that the loop expansion could effectively generate terms that are unsuppressed by the cut-off scale of the (nonrenormalizable) field theory.

On the basis of these severe fine-tuning issues one might be tempted to disregard completely the GPMP test theory. But I propose that, at a strictly phenomenological level of analysis, a fine-tuning problem, however severe, cannot provide sufficient motivation for disregarding a scenario. Actually some of the most successful theories used in fundamental physics are affected by severe fine tuning. Eventually we learn that the fine tuning is only apparent, that some hidden symmetry was actually "naturally" setting up the hierarchy of parameters. And it appears that some symmetry principles could also stabilize the GPMP field theory [41].

So I advocate a viewpoint such that the fine-tuning issues do not cause much concern. But a severe challenge remains: how do we analyze dynamics with such a nonrenormalizable field theory, affected by troublesome UV pathologies? Even if we limit the analysis of the GPMP test theory to tree level, following a strategy which has proven fruitful in other effective-field-theory approaches, one could still wonder whether the tree-level analysis should be limited to dimension-5 operators (as assumed in my description of the GPMP test theory) or one should also include the other types of operators that would be generated through loop effects by those same dimension-5 terms. I propose that, at the present stage of phenomenology work, it is legitimate to focus on the dimension-5 operators. This provides a scenario which can be tested experimentally, and, as I emphasize later in this paper, experiments can test this scenario in some detail in the coming years. So rather than dwell on the specific type of UV sector that would be needed to stabilize the scenario with dimension-5 operators, we can focus our efforts on the relevant phenomenology. If the scenario turns out to be excluded by data the technical issues become irrelevant, and if instead the scenario actually turns out to accommodate nicely some data all the conceptual concerns will be immediately disregarded.

B. On the pure-kinematics phenomenology

I introduced the minimal AEMNS test theory as the natural starting point for a phenomenology that prudently focuses on kinematics, since dynamics is so poorly understood in the relevant quantum-spacetime pictures. While this prudent approach may be attractive from a conceptual perspective, in setting up a phenomenology the idea of focusing on pure-kinematics tests is very challenging. The number of contexts in which dynamics does not have an explicit role is of course very limited, and, even when not immediately evident, a role for dynamics may easily be hidden somewhere deep in the analysis.

I propose to handle this challenge by adopting a rather narrow definition of what a pure-kinematics test should be. There are clearly at least two aspects of particle-physics analyses which are truly a reflection of pure kinematics:
• the structure of the energy-momentum dispersion relation (and the associated relation between speed and energy of the particles) is fixed by pure kinematics, by symmetry principles. By adding new interactions for a given field on may achieve a shift of the mass, but the structure of the dispersion relation is protected by symmetry.
• and symmetries also fix the threshold requirements for particle-physics reaction processes: no matter which type of interactions are introduced, if special-relativistic kinematics is assumed a center-of-mass collision between two photons can produce and electron positron pair only if each photon carries energy larger than the mass of electron.

We can therefore perform a pure-kinematics test if we focus on the analysis of the speed of propagation of particles or we focus on the energy-threshold requirements for particle-physics reaction processes.

Concerning the energy-threshold tests which I will discuss later on a difficulty arises from the fact that sometimes one is not certain about the energies of the incoming particles, and, as I shall stress, in some cases an incorrect identification of the energy of one of the particles could be compensated by a correspondingly incorrect description of

some relevant cross sections. In this specific sense energy-threshold tests may sometimes involve a mix between pure-kinematics and dynamics aspects of the theory. I will attempt to examine this issue in detail in the following. But, at least from a conceptual perspective, it is important to realize that genuine pure-kinematics tests using energy-threshold studies can be performed when all the incoming energies are known. For example, if in some collisions involving two photons both with $0.2 MeV$ ($< m_{electron}$) an electron-positron pair was produced, then special-relativistic kinematics would be ruled out.

IV. EXAMPLES OF PHENOMENOLOGICAL ANALYSES WITH THE TWO TEST THEORIES

The realization that Planck-scale modifications of the dispersion relation could lead to observably large effects [1] has generated a large research effort over these past few years. As mentioned most of this work relied on rather preliminary analyses. The key objective was to demonstrate that indeed Planck-scale sensitivity could be achieved. But now that this issue concerning sensitivity is settled, it is necessary to adopt a more robust style of phenomenology work. In particular, a meaningful comparison of the sensitivities achievable with different types of data must rely on the commonly-adopted language of some reference test theories. It is of course meaningless to compare limits obtained within different test theories. And there is no scientific content in an experimental limit claimed on a vaguely defined test theory. In the recent literature there has been a proliferation of papers claiming to improve limits on Planck-scale modifications of the dispersion relation, but the different studies were simply considering the same type of dispersion relation within significantly different (and sometimes not fully characterized) test theories.

For the reasons discussed in the previous section, the minimal AEMNS test theory and the minimal GPMP test theory may provide a good choice of reference test theories. And their analysis allows to illustrate that the outlook of a certain class of data when examined at the level of a test theory may be very different from what one might expect on the basis of a simplistic sensitivity estimate.

Besides stressing this point concerning test theories, I also want to stress that it is important to be absolutely conservative in assessing the robustness of the data we are confronted with. The fact that most of the data relevant for this quantum-gravity phenomenology comes from astrophysics, and we are therefore not in the comforting situation of repeated controlled experiments, should be a forceful motivator for conservative data analyses.

In this section I will consider certain types of phenomenological analyses that illustrate my proposed strategy of analysis for the two test theories.

A. Derivation of limits from time-of-flight analyses

The best known strategy for establishing experimental limits on Planck-scale modifications of the dispersion relation is based [1] on the fact that both in the AEMNS test theory and in the GPMP test theory one expects a wavelength dependence of the speed of photons, by combining the modified dispersion relation and the relation $v = dE/dp$. At "intermediate energies" ($m < E \ll E_p$) this velocity law will take the form

$$v \simeq 1 - \frac{m^2}{2E^2} + \eta \frac{n+1}{2} \frac{E^n}{E_p^n} \ . \tag{21}$$

Whereas in ordinary special relativity two photons ($m = 0$) emitted simultaneously would always reach simultaneously a far-away detector, according to (21) two simultaneously-emitted photons should reach the detector at different times if they carry different energy. Moreover, in the case of the GPMP test theory even photons with the same energy would arrive at different times if they carry different polarization. In fact, while the minimal AEMNS test theory assumes universality, and therefore a formula of this type would apply to photons of any polarization, in the GPMP test theory, as mentioned, the sign of the effect is correlated with polarization. As a result, while the AEMNS test theory is best tested by comparing the arrival times of particles of different energies, the GPMP test theory is best tested by considering the highest-energy photons available in the data and looking for a sizeable spread in times of arrivals (which one would then attribute to the different speeds of the two polarizations).

This time-of-arrival-difference effect can be significant [1,42] in the analysis of short-duration bursts of photons that reach us from far away sources.

In the near future an opportunity to test this effect will be provided by observations of gamma-ray bursters. For a gamma-ray burst it is not uncommon that the time travelled before reaching our Earth detectors be of order $T \sim 10^{17} s$. And microbursts within a burst can have very short duration, as short as $10^{-3} s$ (or even $10^{-4} s$), and this means that the photons that compose such a microburst are all emitted at the same time, up to an uncertainty of $10^{-3} s$. Some

of the photons in these bursts have energies that extend at least up to the GeV range. For two photons with energy difference of order $\Delta E \sim 1 GeV$ a $\eta \Delta E/E_p$ speed difference over a time of travel of $10^{17}s$ would lead to a difference in times of arrival of order

$$\Delta t \sim \eta T \Delta \frac{E}{E_p} \sim 10^{-2}s \ , \tag{22}$$

which is significant (the time-of-arrival differences would be larger than the time-of-emission differences within a single microburst).

For the AEMNS test theory the Planck-scale-induced time-of-arrival difference could be revealed [1,42] upon comparison of the "average arrival time" of the gamma-ray-burst signal (or better a microburst within the burst) in different energy channels. The GPMP test theory would be most effectively tested by looking for a dependence of the time-spread of the bursts that grows with energy (at low energies the effect is anyway small, so the polarization dependence is ineffective, whereas at high energies the effect may be nonnegligible and an overall time-spread of the burst could result from the dependence of speed on polarization).

Since the quality of relevant gamma-ray-burst data is still relatively poor, the present best limit was obtained in Ref. [42]: the negative results of a search of time-of-arrival/energy correlations for a TeV-gamma-ray short-duration flare from the Markarian 421 blazar allowed to deduce the limit $|\eta| < 3 \cdot 10^2$. Assuming that the relevant gamma-ray emission was not largely polarized, one would correspondingly obtain for the minimal GPMP test theory $|\eta_\gamma| < 1.5 \cdot 10^2$ (the factor-2 difference in sensitivities for $|\eta|$ and for $|\eta_\gamma|$ is due to the fact that there is a $2|\eta_\gamma|$ speed-difference effect between polarizations). However, to my knowledge, the possibility of large polarization of the relevant gamma-ray emission (while unexpected) is not excluded by data, and therefore for the minimal GPMP test theory these data do not allow to establish a fully robust limit.

The sensitivities achievable [53] with the next generation of gamma-ray telescopes, such as GLAST [53], could allow to test very significantly (21) in the case $n = 1$, by possibly pushing the limit on η far below 1 (whereas the effects found in the case $n = 2$, $|\eta| \sim 1$ are too small for GLAST). Whether or not these levels of sensitivity to the Planck-scale effects are actually achieved may depend on progress in understanding other aspects of gamma-ray-burst physics. In fact, the Planck-scale-effect analysis would be severely affected if there were poorly understood at-the-source correlations between energy of the photons and time of emission. In the recent Ref. [54] it was emphasized that it appears that one can infer such an energy/time-of-emission correlation from available gamma-ray-burst data. The studies of Planck-scale effects will be therefore confronted with a severe challenge of "background/noise removal". At present it is difficult to guess whether this problem can be handled successfully. We do have a good card to play in this analysis: the Planck-scale picture predicts that the times of arrival should depend on energy in a way that grows in exactly linear way with the distance of the source. One may therefore hope that, once a large enough sample of gamma-ray bursts (with known source distances) becomes available, one might be able disentangle the Planck-scale propagation effect from the at-the-source background.

An even higher sensitivity to possible Planck-scale modifications of the velocity law could be achieved by exploiting the fact that, according to current models [55], gamma-ray bursters should also emit a substantial amount of high-energy neutrinos. Some neutrino observatories should soon observe neutrinos with energies between 10^{14} and 10^{19} eV, and one could, for example, compare the times of arrival of these neutrinos emitted by gamma-ray bursters to the corresponding times of arrival of low-energy photons. One could use this strategy to test rather stringently[11] the case of (21) with $n = 1$, an even perhaps gain some access to the investigation of the case $n = 2$.

In order to achieve these sensitivities with neutrino studies once again some technical and conceptual challenges should be overcome. Also this type of analysis requires an understanding of gamma-ray bursters good enough to establish whether there are typical at-the-source time delays. The analysis would loose much of its potential if one cannot exclude some systematic tendency of gamma-ray bursters to emit high-energy neutrinos with, say, a certain delay with respect to microbursts of photons. But also in this case one could hope to combine several observations from gamma-ray bursters at different distances in order to disentangle the possible at-the-source effect.

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[11]Note however that in an analysis mixing the properties of different particles the sensitivity that can be achieved will depend strongly on whether universality of the modification of the dispersion relation is assumed. For example, for the GPMP test theory a comparison of times of arrival of neutrinos and photons could only introduce a bound on some combination of the dispersion-relation-modification parameters for the photon and for the neutrino sectors.

B. Analysis of threshold-energy requirements in the laboratory

As mentioned, in addition to the possible manifestation in time-of-arrival/energy correlations, the quantum-gravity-scale modifications of the dispersion relation could have observably-large implications for what concerns the analysis of energy threshold for particle-physics reactions. This possibility has been studied primarily in contexts of interest in astrophysics [43–46], where the relevant scales turn out to be favourable for achieving high sensitivity. But before commenting on those analyses in astrophysics, I find useful to consider the study of these processes from a gedanken-experiment perspective, imagining to perform analogous studies in the controlled environment of a laboratory setup.

I should first of all emphasize that I will focus on the possibility that the modification of the dispersion relation occurs in a framework in which the law of energy-momentum conservation is not modified. Both the AEMNS test theory and the GPMP test theory involve modified dispersion relations and unmodified laws of energy-momentum conservation (the fact that the law of energy-momentum conservation is not modified is explicitly among the ingredients of the AEMNS test theory, while in the GPMP test theory it follows from the adoption of low-energy effective field theory).

In this paper I am not discussing in detail the case of modified dispersion relations introduced within a "doubly-special relativity" scenario [8,9,56], which I already mentioned in the discussion of κ-Minkowski spacetime. I am in fact here focusing on scenarios for broken Lorentz symmetry (rather than deformed Lorentz symmetry). Test theories for doubly-special relativity scenarios with modified dispersion relations are under consideration (see, e.g., Ref. [57,58]), but I will not make room for them here. It is appropriate however to stress in this subsection that the assumption of modified dispersion relations and unmodified laws of energy-momentum conservation is inconsistent with the doubly-special relativity principles, since it inevitably [8] gives rise to a preferred class of inertial observers. A doubly-special relativity scenario with modified dispersion relations must necessarily have a modified law of energy-momentum conservation.

Going back to the AEMNS and GPMP test theories which I am considering, in this subsection I want to stress that combining a modified dispersion relation with unmodified laws of energy-momentum conservation one naturally finds a modification of the threshold requirements for certain reactions. Let us in particular consider the dispersion relation (12), with $n = 1$, for the AEMNS test theory, in the analysis of a process $\gamma\gamma \to e^+e^-$, a collision between a soft photon of energy ϵ and a high-energy photon of energy E which might produce an electron-positron pair. For given soft-photon energy ϵ, the process $\gamma\gamma \to e^+e^-$ is allowed only if E is greater than a certain threshold energy E_{th} which depends on ϵ and m_e^2. For $n = 1$, combining (12) with unmodified energy-momentum conservation, this threshold energy (assuming $\epsilon \ll m_e \ll E_{th} \ll E_p$) must satisfy

$$E_{th}\epsilon + \eta\frac{E_{th}^3}{8E_p} \simeq m_e^2 \ . \tag{23}$$

The special-relativistic result $E_{th} = m_e^2/\epsilon$ corresponds of course to the $\eta \to 0$ limit of (23). For $|\eta| \sim 1$ the Planck-scale correction can be safely neglected as long as $\epsilon > (m_e^4/E_p)^{1/3}$. But eventually, for sufficiently small values of ϵ (and correspondingly large values of E_{th}) the Planck-scale correction cannot be ignored.

This is another pure-kinematics test: if a $10TeV$ photon collides with a photon of $0.03eV$ and produces an electron-positron pair the case $n = 1$, $\eta \sim -1$ for the AEMNS is ruled out. A $10TeV$ photon and a $0.03eV$ photon can produce an electron-positron pair according to ordinary special-relativistic kinematics (and its associated requirement $E_{th} = m_e^2/\epsilon$), but they cannot produce an electron-positron pair according to AEMNS kinematics (and its associated requirement (23)).

So for negative η the AEMNS test theory could be ruled relying on pure-kinematics data. For positive η the situation is somewhat different. While negative η increases the energy requirement for electron-positron pair production, positive η decreases the energy requirement for electron-positron pair production. In some cases where one would expect electron-positron pair production to be forbidden the AEMNS test theory with positive η would instead allow it. But once a process is allowed there is no guarantee that it will actually occur, not without some information on the description of dynamics (that allows us to evaluate cross sections). So, from a fully conservative perspective, a pure-kinematics framework can be falsified when it predicts that a process cannot occur (if instead the process is seen) but it cannot be falsified when it predicts that a process is allowed.

In the case of the minimal GPMP test theory the availability of the field-theoretic setup for the description of dynamics renders this types of studies even more powerful (when the GPMP predicts that a process is allowed it also predicts the relevant probability amplitudes). Also for tests of the minimal GPMP test theory in the laboratory one could use a beam of $10TeV$ photon and a beam of $0.03eV$ photons. However, for the GPMP test theory it would be very useful to control helicity/polarization of the beams.

It is actually not so inconceivable [46] to conduct this type of tests really in a laboratory (since the production of 10-TeV photons is not so far from our present technical capabilities), but this might have to wait a few years. In the

meantime some contexts in astrophysics provide us an opportunity to study the relevant processes, although rather indirectly and without the comfort level of a controlled laboratory setup.

C. Limits obtained from observed absorption of TeV photons from Blazars

The type of threshold-energy scenario discussed in the preceding subsection could have [43–46] observably-large implications for what concerns the opacity of our Universe to various types of high-energy particles. Of particular interest is the fact that, according to the conventional (classical-spacetime) description, the infrared diffuse extragalactic background should give rise to strong absorption of "TeV photons" (here understood as photons with energy $1TeV < E < 30TeV$). The relevant process is of course $\gamma\gamma \to e^+e^-$, already analyzed in the preceding subsection.

If the photon of energy ϵ is part of the infrared diffuse extragalactic background and the photon emitted by a blazar is of TeV-range energy one finds that the prediction for absorption of the hard photon by the infrared diffuse extragalactic background can be significantly modified.

The classical-spacetime analysis, in which a key role is played by the threshold condition $\epsilon \geq m_e^2/E$, the distance of the blazar, and the density of the infrared diffuse extragalactic background, leads to a prediction of the amount of absorption to be expected as a function of the energy of the photons emitted by the blazar. The experimental verification of this classical-spacetime prediction has made significant progress over the last couple of years: evidence of absorption of TeV photons has been reported in observations [59,60] of the Markarian 421 blazar (at a redshift of $z = 0.031$), in observations [61] of the Markarian 501 blazar (at a redshift of $z = 0.034$), and in observations [62] of the blazar H1426+428 (at a redshift of $z = 0.129$). While these observations all concerned γ-rays up to energies in the 20-TeV range, observations of γ-rays up to 45 TeV from Markarian 421 have been recently reported [63], and again the data are found [63] to reflect significant absorption.

Perhaps the most convincing evidence of TeV-photon absorption come from the analysis of the combined X-ray/TeV-gamma-ray spectrum for the Markarian 421 blazar, as discussed in particular in Ref. [64]. The X-ray part of the spectrum allows to predict the TeV-gamma-ray part of the spectrum in a way that is rather insensitive on our poor knowledge of the source. This in turn allows to establish in a source-independent way that absorption is occurring.

The fact that the observations still give us only a preliminary picture of absorption together with the fact that there is a significant level of uncertainty in phenomenological models of TeV blazars and in phenomenological models of the density of the infrared diffuse extragalactic background does not allow us to convert these observations into tight limits on departures from the classical-spacetime analysis. However, I intend to argue that even just the basic fact that we see absorption of TeV γ-rays allows to derive a rather robust limit.

Previous studies (see, *e.g.*, Refs. [44,65,67]) had already shown that this type of observations, if found to be in agreement with the conventional classical-spacetime picture, could constrain very significantly several models of departure from Lorentz symmetry. In line with these previous studies, and using the fact that the observations recently reported in Ref. [63] further extend the energy range of observations of TeV blazars, one can easily verify that Planck-scale sensitivity (intended as $\eta \sim 1$ sensitivity, at least for $n = 1$) is within reach for the near future. However, my main focus is not on this type of future-sensitivity estimate, but rather on the limit that can be conservatively/robustly established using presently-available information, *i.e.* using the bare fact that some absorption of TeV γ-rays is evident in the data.

In fact, while the presence of some level of absorption of TeV γ-rays is indeed evident in the observations reported in Refs. [59–63], these observations are still insufficient to make a quantitative comparison with the predictions of the classical-spacetime picture, at least not consistently with the prudently conservative attitude one must adopt in attempting to establish an unconditional limit on parameters such as η. The fact that some absorption of TeV γ-rays is being seen can be robustly inferred from the structure of all of the observations reported in Refs. [59–63]. On the other hand if one looks in detail at the information that is emerging from these observations it is rather clear that we are not ready for stating robustly that the predictions of the classical-spacetime picture are finding detailed confirmation:

(j) Some authors have discussed [59–61,68] a puzzling difference between the cutoff energy found in data concerning Markarian 421, $E_{mk421}^{cutoff} \simeq 3.6TeV$, and the corresponding cutoff estimate obtained from Markarian 501, $E_{mk501}^{cutoff} \simeq 6.2TeV$, a difference which appears to be significant at the 3σ level. Since Markarian 421 and Markarian 501 are at comparable distances from the Earth (at redshifts of $z = 0.031$ and $z = 0.034$ respectively) and they are expected to host very similar mechanisms of emission of TeV γ-rays, this difference in the estimated cutoff energy may be an indication that we do not yet have a robust picture of what is going on.

(jj) The observation of TeV γ-rays emitted by the blazar H1426+428, which is at a redshift four times bigger than the one of Markarian 421 and Markarian 501, does show, as expected in the standard picture, a level of absorption

which is higher than the ones inferred for Markarian 421 and Markarian 501. However, as emphasized in Ref. [62], even taking into account the uncertainties on the density of the infrared diffuse extragalactic background, "the TeV luminosity seems to exceed the level anticipated from the current models of TeV blazars by far" [62].

(jjj) As mentioned, a detailed comparison of observed absorption with corresponding predictions of the classical-spacetime (Lorentz-invariant) description of absorption by the infrared diffuse extragalactic background of γ-rays emitted by blazars would require correspondingly accurate descriptions of the spectrum emitted by the blazars and of the density of the infrared diffuse extragalactic background. However, measurements of the density of the infrared diffuse extragalactic background are very difficult and as a result our experimental information on this density is still affected by large uncertainties [68,69]. Similarly, there are models of TeV blazars which appear to be rather robust theoretically, but some of the above-mentioned observational facts (the different cutoff estimates for Markarian 421 and Markarian 501 and the unexpectedly large TeV luminosity of the H1426+428 blazar) impose us to treat cautiously the indications obtained from these theoretical models.

These points (j), (jj), (jjj) impose us to analyze prudently the implications of the observations reported in Refs. [59–63]. I shall not assume that the observations imply any level of agreement with the classical spacetime picture, but I will insist that the Planck-scale effect be consistent with the fact, now established, that TeV γ-rays with energies up to 20 TeV are absorbed by the infrared diffuse extragalactic background. This suggests that at least some photons with energy smaller than $\sim 200 meV$ can create an electron-positron pair in collisions with a $20 TeV$ γ-ray. For the AEMNS test theory, in light of Eq. (23), this observation leads to

$$\eta \geq -46 \tag{24}$$

(*i.e.* either η is positive or η is negative with absolute value smaller than 46).

Other authors [65–67] have argued that a certain level of agreement with the predictions of the classical-spacetime picture can be inferred from the data, and on that basis they have derived more stringent limits than the one I am claiming in (24). However, I am here insisting on a concept of experimental limit that is absolutely conservative, an experimental limit that can be truly considered as an unavoidable fact to be taken into account by theorists. For the reasons discussed above any claim that there is some agreement in the observed absorption and the level of absorption predicted by the classical-spacetime picture would be <u>conditional</u> to the success of some, still unproven, models of TeV-gamma-ray emission by blazars and of the infrared diffuse extragalactic background.

Actually, one might argue that even the more prudent limit (24) I am advocating should be subject to further scrutiny. In fact, we see absorption of the multi-TeV gamma rays and we assume it should be due to interactions with infrared photons; however, one could conjecture that perhaps the absorption is due to higher-energy background photons. Since we intend to confine the analysis of the AEMNS test theory to the realm of pure kinematics, any bound that is established on the parameters of the AEMNS test theory should be completely insensitive on dynamics issues. We should therefore contemplate the possibility that the AEMNS kinematics be implemented within a framework in which the description of dynamics is such to introduce a large-enough modification of cross sections to allow absorption of multi-TeV blazar gamma rays by background photons of energy higher than $200 meV$. This would be a way to evade the bound (24). Consistently with the absolutely conservative approach I am advocating I will therefore not describe (24) as a fully-established experimental bound. I propose however that (24) is rather robust: the only way to evade it requires a sort of careful "conspiracy" of new effects. In the standard picture photons with below-TeV energy should not be absorbed while multi-TeV photons should be absorbed by infrared photons. This picture is of course also consistent with AEMNS kinematics, if η satisfies the requirement (24). In order to allow negative values of η of absolute value larger than prescribed by (24) one would need a corresponding modification of cross sections, so that the modification of kinematics and the modification of cross sections would conspire to leave the numerical value of the scale of onset of absorption basically unchanged with respect to the standard picture. It is hard to believe that such a conspiracy would be in place, but I will still describe the limit (24) as established "up to conspiracies".

So far in this subsection my derivation and discussion of the limit (24) is strictly applicable only to the minimal AEMNS test theory. For the case of the minimal GPMP test theory the analysis is simplified by the fact that one has the field-theoretic setup for the evaluation of cross sections, but on the other hand one has the complication of having to take into account the fact that the modification of the dispersion relation carries opposite sign for the two polarizations of the photon and for the two helicities of the electron/positron. And one should also take into account that, while observations now provide robust evidence of some absorption of TeV gamma rays, a conservative phenomenological analysis should (as a result of the mentioned residual grey areas in the understanding of these observations) consider the possibility that only one of the polarizations is being absorbed. I postpone this more involved analysis to a future study.

D. Derivation of limits from analysis of UHE cosmic rays

In the preceding subsection I discussed the implications of possible Planck-scale effects for the process $\gamma\gamma \to e^+e^-$, but of course this is not the only process in which Planck-scale effects can be important. In particular, there has been strong interest [43–46,67,70,71,80] in "photopion production", $p\gamma \to p\pi$, where again the combination of (12) with unmodified energy-momentum conservation leads to a modification of the minimum proton energy required by the process (for given photon energy). In the case in which the photon energy is the one typical of CMBR photons, in the AEMNS test theory one finds that the threshold proton energy can be significantly shifted upward (for negative η), and this in turn should affect at an observably large level the expected "GZK cutoff" for the observed cosmic-ray spectrum. Observations reported by the AGASA [81] cosmic-ray observatory provide some encouragement for the idea of such an upward shift of the GZK cutoff, but the issue must be further explored. Forthcoming cosmic-ray observatories, such as Auger [82], should be able [43,46] to fully investigate this possibility.

Of course, also for the cosmic-ray GZK threshold, just like for the gamma-ray absorption threshold discussed in the preceding subsection, the AEMNS analysis should contemplate the possibility of a "conspiracy", although in this case it appears to be an ubelievable conspiracy. If the only background radiation available for photopion production was the CMBR, then the prediction of an upward shift of the GZK cosmic-ray cutoff within the AEMNS test theory, for negative η, would be completely robust. But background radiation has many components and one could again (as in the case of the gamma-ray absorption threshold) contemplate the possibility to combine AEMNS kinematics with an unspecified description of dynamics such that interactions of cosmic rays with other components of the background radiation would lead to a net result that does not change the numerical value of the GZK threshold. At least for $n = 1$ and negative η of order 1, I advocate that this "conspiracy scenario" should be dismissed. For $n = 1$ and negative η of order 1 the AEMNS kinematics allows the interaction of cosmic rays only with photons of energy higher than the TeV scale (see Ref. [46]). The density of such high energy background photons is extremely low, and therefore, even in a prudent phenomenology, this "conspiracy scenario" can indeed be dismissed.

For the minimal GPMP test theory this issue of possible conspiracies is of course absent, since the field-theoretic setup allows to evaluate cross sections, but one must take into account that for one of the helicities of the proton the dispersion relation is of negative-η type while for the other helicity the dispersion relation is of positive-η type. One would then expect roughly one half of the UHE protons to evade the GZK cutoff, so the cutoff would still be violated but in a softer way than in the case of the AEMNS test theory with negative η.

E. Derivation of limits from analysis of photon stability

The cases considered in the preceding two subsections, the one of TeV-gamma-ray photon absorption and the one of photopion production, are examples of situations in which a given process is allowed in presence of exact Lorentz symmetry but can be kinematically forbidden in presence of certain departures from Lorentz symmetry. The opposite is also possible: some processes that are kinematically forbidden in presence of exact Lorentz symmetry become kinematically allowed in presence of certain departures from Lorentz symmetry.

Certain observations in astrophysics, which allow us to establish that photons of energies up to $\sim 10^{14} eV$ are not unstable, can be particularly useful [67,70–72] in setting limits on some schemes for departures from Lorentz symmetry. Let us for example analyze the process $\gamma \to e^+e^-$ from the AEMNS perspective, using the dispersion relation (12), with $n = 1$, and unmodified energy-momentum conservation. One easily finds a relation between the energy E_γ of the incoming photon, the opening angle θ between the outgoing electron-positron pair, and the energy E_+ of the outgoing positron (of course the energy of the outgoing electron is simply given by $E_\gamma - E_+$). For the region of phase space with $m_e \ll E_\gamma \ll E_p$ this relation takes the form

$$\cos(\theta) \simeq \frac{E_+(E_\gamma - E_+) + m_e^2 - \eta E_\gamma E_+(E_\gamma - E_+)/E_p}{E_+(E_\gamma - E_+)} \ , \tag{25}$$

where m_e is the electron mass.

The fact that for $\eta = 0$ Eq. (25) would require $cos(\theta) > 1$ reflects the fact that, if Lorentz symmetry is preserved, the process $\gamma \to e^+e^-$ is kinematically forbidden. For $\eta < 0$ the process is still forbidden, but for positive η high-energy photons can decay into an electron-positron pair. In fact, for $E_\gamma \gg (m_e^2 E_p/|\eta|)^{1/3}$ one finds that there is a region of phase space where $\cos(\theta) < 1$, *i.e.* there is a physical phase space available for the decay.

The energy scale $(m_e^2 E_p)^{1/3} \sim 10^{13} eV$ is not too high for testing, since, as mentioned, in astrophysics we see photons of energies up to $\sim 10^{14} eV$ that are not unstable (they clearly travel safely some large astrophysical distances).

Within AEMNS kinematics, for $n = 1$ and positive η of order 1, it would have been natural to expect that such photons with $\sim 10^{14} eV$ energy would not be stable. Once again, before claiming that $n = 1$ and positive η of order

1 is ruled out, one should be concerned about possible conspiracies. The fact that the decay of $10^{14}eV$ photons is allowed by AEMNS kinematics (for $n = 1$ and positive η of order 1) of course does not guarantee that these photons should rapidly decay. It depends on the relevant probability amplitude, whose evaluation goes beyond the reach of kinematics. I am unable to provide an intuition for how big of a conspiracy would be needed to render $10^{14}eV$ photons stable compatibly with AEMNS kinematics with $n = 1$ and $\eta = 1$. My tentative conclusion is that $n = 1$ with positive η of order 1 is ruled out "up to conspiracies", but unlike the case of the GZK-threshold analysis I am unprepared to argue that the needed conspiracy is truly unbelievable.

For the GPMP test theory the photon stability analysis is weakened because of other reasons. There one does have the support of the effective-field-theory descrition of dynamics, and within that framework one can exclude huge suppression by Planck scale effects of the interaction vertex needed for $\gamma \to e^+e^-$ around $\sim 10^{13}eV$, $\sim 10^{14}eV$. So the limit-setting effort is not weakened by the absence of an interaction vertex. However, as mentioned, consistency with the effective-field-theory setup requires that the two polarizations of the photon acquire opposite-sign modifications of the dispersion relation. We observe in astrophysics some photons of energies up to $\sim 10^{14}eV$ that are stable over large distances, but as far as we know those photons could be all, say, right-circular polarized (or all left-circular polarized). I postpone a detailed analysis to future work, but let me note here that[12] there is a region of minimal-GPMP parameter space where both polarizations of a $\sim 10^{14}eV$ photon are unstable (a subset of the region with $|\eta_f| > |\eta_\gamma|$). That region of parameter space is of course excluded by the photon-stability data.

F. Derivation of limits from analysis of synchrotron radiation

A recent series of papers [73–77,48,78] has focused on the possibility to set limits on Planck-scale modified dispersion relations focusing on their implications for synchrotron radiation. By comparing the content of the first estimates[13] produced in this research line [73] with the understanding that emerged from follow-up studies [74–77,48,78] one can gain valuable insight on the risks involved in analyses based on simplistic order-of-magnitude estimates, rather than careful comparison with meaningful test theories. In Ref. [73] the starting point is the observation that in the conventional (Lorentz-invariant) description of synchrotron radiation one can estimate the characteristic energy E_c of the radiation through a heuristic analysis [79] leading to the formula

$$E_c \simeq \frac{1}{R \cdot \delta \cdot [v_\gamma - v_e]} \ , \tag{26}$$

where v_e is the speed of the electron, v_γ is the speed of the photon, δ is the angle of outgoing radiation, and R is the radius of curvature of the trajectory of the electron.

Assuming that the only Planck-scale modification in this formula should come from the velocity law (described using $v = dE/dp$ in terms of the modified dispersion relation), one finds that in some instances the characteristic energy of synchrotron radiation may be significantly modified by the presence of Planck-scale departures from Lorentz symmetry. As an opportunity to test such a modification of the value of the synchrotron-radiation characteristic energy one can hope to use some relevant data [73,75] on photons detected from the Crab nebula. This must be done with caution since the observational information on synchrotron radiation being emitted by the Crab nebula is rather indirect: some of the photons we observe from the Crab nebula are attributed to sychrotron processes on the basis of a rather successful model, and the value of the relevant magnetic fields is also not directly measured.

Assuming that indeed the observational situation has been properly interpreted, and relying on the mentioned assumption that the only modification to be taken into account is the one of the velocity law, one could basically rule out [73] the case $n = 1$ with negative η for a modified dispersion relation of the type (12).

This observation led at first to some excitement, but more recent papers are starting to adopt a more prudent viewpoint. The lack of comparison with a meaningful test theory represents a severe limitation of the original analysis. In particular, synchrotron radiation is due to the acceleration of the relevant electrons and therefore implicit in the derivation of the formula (26) is a subtle role for dynamics [74]. From a field-theory perspective the process

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[12]I thank an anonimous referee for bringing this point to my attention.

[13]Ref. [73] is at this point obsolete, since the relevant manuscript has been revised for the published version [75] and the recent Ref. [78] provides an even more detailed analysis. It is nevertheless useful to consider this series of manuscripts [73,75,78] as an illustration of how much the outlook of a phenomenological analysis may change in going from the level of simplistic order-of-magnitude estimates to the level of careful comparison with meaningful test theories.

of synchrotron-radiation emission can be described in terms of Compton scattering of the electrons with the virtual photons of the magnetic field. One would therefore be looking deep into the dynamical features of the theory.

The minimal AEMNS test theory does assume a modified dispersion relation of the type (12) universally applied to all particles, but it is a pure-kinematics framework and, since the analysis involves some aspects of dynamics, it cannot be tested using a Crab-nebula synchrotron-radiation analysis. I have stressed that also in other instances, like the analysis of the cosmic-ray GZK threshold and the analysis of the gamma-ray absorption threshold, there is a possible (conspiracy-type) hidden role of dynamics in the AEMNS analysis, but in this synchrotron-radiation context the role of dynamics is explicit and unavoidable. For example, the concerns about dynamics in the analysis of the gamma-ray absorption threshold are only an accident due to the fact that we have no control over the radiation background. If one could set up controlled laboratory collisions between $10TeV$ photons and $0.03eV$ photons, then a pure-kinematics analysis could be truly performed, without any risk of "contamination" from dynamics. Instead even the study of synchrotron radiation in a controlled laboratory setup could not be used as a pure-kinematics test: in the laboratory we can switch on the electron beam and the external magnetic field, but we have no control on the description/nature of virtual particles.

For what concerns the minimal GPMP test theory, where the dynamical aspects of the problem are handled according to the field-theoretic setup, the usefulness of this Crab-nebula synchrotron-radiation analysis is reduced by the fact that we do not know whether both helicities of the electron (or positron) are contributing to the synchrotron-radiation emission. Through the Crab-nebula synchrotron-radiation analysis one therefore obtains no constraint on the minimal GPMP test theory. The Crab-nebula synchrotron-radiation analysis can however, as stressed in Ref. [78], introduce a valuable constraint on more general formulations of the GPMP test theory, in which one accommodates independent free parameters for the dispersion-relation modifications of the two helicities of a fermion.

V. SOME OTHER OPPORTUNITIES TO CONSTRAIN PARAMETERS OF THE TEST THEORIES

As stressed earlier, the primary objective of this paper is the one of igniting a transition to a new more mature phase of quantum-gravity phenomenology, in which different works are compared in terms of the common language of some reference test theories. I discussed two "minimal" test theories that could be considered for this purpose, and, in the preceding section, I illustrated the type of issues that can arise in working with these two test theories in the context of a few among the most popular opportunities to test Planck-scale departures from Lorentz symmetry. While for my purposes it was sufficient to discuss a few examples of analyses of the test theories, without attempting to provide a "status report" on the absolute best limits achievable with presently-available data, in this section I do want to comment briefly on some other possible opportunities to constrain the two minimal test theories.

Let me start by mentioning that both the AEMNS and GPMP test theories are preferred-frame theories, and their consistency with the relevant classic tests (*e.g.* Hughes-Drever tests) should be examined. Those working in the field have always quickly assumed that, because of the low energies of the particles involved in those tests, the corresponding experimental bounds would be of marginal significance. This type of low-energy tests usually constrains more effectively deformations that are not suppressed at low energies (*e.g.* deformations that in field-theory language correspond to dimension-4 operators, rather than the dimension-5 operators of the GPMP test theory). However, since the field has now reached a certain maturity, it appears that a careful analysis of these "preferred-frame tests" should be among the priorities for future developments.

For what concerns specifically the parameter η_f of the minimal GPMP test theory, it has been argued in Ref. [49] that through the results of measurements of spin-polarized torsion-pendulum frequencies [83] one can establish $|\eta_f| \leq 2$.

Concerning particle decays I should mention that, while in some cases departures from Lorentz symmetry allow the decay of stable particles (as in the discussed $\gamma \to e^+e^-$ context), it is also possible for departures from Lorentz symmetry of the type codified in the minimal AEMNS and minimal GPMP test theories to render stable, at ultra-high energies, a particle which would be unstable in the standard framework. In particular, there has been some interest [70,71] in the possibility that the process $\pi \to \gamma\gamma$ might be forbidden at ultrahigh energies.

Another opportunity that has generated interest recently and I have not mentioned so far, is the one of the vacuum Cerenkov constraint, analyzed in the sense considered for example in Ref. [78]. This is particularly of interest for some generalizations of the minimal GPMP test theory, where the vacuum-Cerenkov constrain and the synchrotron-radiation constrain can be considered in an overall analysis [78] of Crab-nebula data. The effectiveness of this overall analysis may be reduced by the fact that, as acknowledged in the published version of Ref. [78], one must consider even the possibility that the Crab-nebula synchrotron radiation be due to positron acceleration, but the analysis is valuable nonetheless [78].

For the GPMP test theory[14] perhaps the best opportunity to constrain the parameter η_γ comes from birefringence analyses: according to the minimal GPMP test theory electromagnetic waves of opposite helicity should have different phase velocities [3,78,84]. As the electromagnetic wave travels its linear polarization should rotate direction as a linear function of time travelled. Experimental limits on this effect can be derived using observations of polarized light from distant galaxies [78,84,85]. The analysis reported in Ref. [84] leads to a very significant limit of $|\eta_\gamma| < 2 \cdot 10^{-4}$. A even more significant limit on the η_γ parameter could be inferred from observation of polarized gamma rays from distant astrophysical sources. One such observation has been recently reported in the literature: Ref. [86] reports polarized MeV gamma rays in the prompt emission of the gamma-ray burst GRB021206. As observed in Ref. [78] this would allow to establish an impressive limit on η_γ ($\eta_\gamma < 10^{-14}$ or even better). However, the report of Ref. [86] has been challenged (see *e.g.* Ref. [87]), and as long as the experimental situation remains unclear of course these data cannot be used to establish robust experimental limits.

VI. CLOSING REMARKS

With this paper I am hoping to ignite a debate which should lead to the transition toward a more mature phase of quantum-gravity phenomenology, in which a key role is played by some reference test theories. I gave an explicit formulation of two test theories which could be considered for this role, and I discussed a few examples of phenomenological analyses with these two test theories. The two test theories assume basically the same type of modification of the dispersion relation, but in my illustrative examples of phenomenological analyses it emerged that the phenomenology is in some cases very different. This exposes the shortcomings of an approach to the phenomenology of Planck-scale modified dispersion relations which had become fashionable in the recent literature: there have been several papers claiming to improve limits on Planck-scale modifications of the dispersion relation, but the different studies were simply considering the same type of dispersion relation within significantly different test theories, or worse the phenomenological analysis did not even rely on a well-defined test theory. From outside the quantum-gravity-phenomenology community these papers were actually perceived as a gradual improvement in the experimental bounds on the overall idea of Planck-scale departures from Lorentz symmetry, to the point that there is now a wide-spread perception that in general departures from Lorentz symmetry are already experimentally constrained to be far beyond the Planck-scale. Instead I showed that two simple and rather natural test theories evade automatically some of the possible opportunities for constraints.

The two test theories on which I focused, the minimal AEMNS test theory and the minimal GPMP test theory, could be rather natural starting points for the two types of intuitions that are being discussed in the quantum-gravity-phenomenology literature. The key point is whether we should trust effective low-energy field theory as the formalism used in the description of dynamical effects. The fact that both in the study of noncommutative spacetimes and in the study of Loop Quantum Gravity, the two quantum pictures of spacetime that provide the key sources of motivation for research on Planck-scale modifications of the dispersion relation, we are really only starting to understand some aspects of kinematics, but we are still missing any robust result on dynamics, encourages an approach to phenomenology which is correspondingly prudent with respect to the description of dynamics. Our test theories will be really successful only if they work well in bridging the gap between experimental data and our present limited understanding of fundamental quantum-gravity/quantum-spacetime pictures. We therefore need a set of test theories reflecting the different intuitions that are guiding different approaches to the quantum-gravity problem. For those who are most concerned about the status of the description of dynamics in quantum-gravity research, the assumption of a description of dynamics based on effective low-energy field theory appears to be too unreliable, and the pure-kinematics minimal AEMNS test theory may provide a natural starting point for phenomenology. The fact that the minimal AEMNS test theory does not assume anything about dynamics of course limits its applicability, but it allows us to focus (at least in this first stage of investigation) on the assumption of universality of the modifications of the dispersion relation. This assumption is in fact fully consistent with the kinematic structure of the test theory, and may well turn out to be also consistent with the description of dynamics (when established), if this description is not field-theory based.

For those who are willing to set aside the concerns about the description of dynamics, and go ahead with the effective-field-theory formulation, the minimal GPMP test theory should provide a valuable starting point. The fact that this test theory can be compared also to data involving some aspects of dynamics obviously allows a richer

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[14]Clearly the minimal AEMNS test theory, with its universal modification of the dispersion relation, predicts no birefringence effects.

phenomenology, but the complication of nonuniversality of the effects must necessarily be accommodated. In fact the effective-field-theory formulation automatically requires that the two polarizations of photons carry opposite-sign modifications of the dispersion relation, and then a natural criterion (in which the speed-of-light scale preserves at least its role in the description of the average speed of randomly-composed particle bursts) leads to assuming the same sign/helicity correlation also for all other particles.

As illustrated by the few examples of phenomenological analyses which I discussed, the phenomenology work with the minimal GPMP test theory is of a rather familiar type. It is a setup that resembles closely the one of certain nonrenormalizable effective low-energy field theories used in particle-physics phenomenology (although, as mentioned, the fine-tuning concerns are more severe). Instead the minimal AEMNS test theory, as it is conceived as a pure-kinematics test theory, will force us to a type of phenomenology which (to my knowledge) is new. The abstract idea of a pure-kinematics test theory is well motivated by the status of our understanding of the relevant quantum-spacetime frameworks, but, as illustrated by the few examples of phenomenological analyses which I discussed, the practical realizations of a pure-kinematics phenomenological analysis are often confronted with the problem of "contamination" by dynamical effects. I structured this paper in such a way that, on this crucial point of the possibility of pure-kinematics analyses, a certain hierarchy would emerge for the reader. At the top of this hierarchy there are some phenomenological analyses that truly involve pure kinematics, like the time-of-flight analyses discussed in subsection IV.A. Then I considered the analyses of the photon-absorption threshold and of the cosmic-ray GZK threshold are examples of phenomenological analyses that could be used for pure-kinematics tests in the controlled environment of a laboratory, where one could control the energies of the colliding particles, but are subject to a "conspiracy hypothesis" in the context of certain applications in astrophysics, where the incoming-particle energy is known but its potential targets have energies that spread over a large range. In those astrophysical applications a conspiracy between the adopted deformation of kinematics and the unspecified deformation of dynamics could affect the reliability of the analysis. And finally there are cases like the one of the synchrotron-radiation analysis, which even in the controlled laboratory setup could not be viewed as tests of pure kinematics. The cases like the synchrotron-radiation analysis, which even in principle is not structured as a pure-kinematics test, are clearly ill-suited for the analysis of a pure-kinematics test theory. More subject to debate is the handling of the analyses which are subject to a "conspiracy hypothesis": while I am advocating a prudent conservative approach to the derivation of experimental bounds in this phenomenology, I have argued that, at least for situations like the one of the AEMNS test theory with $n = 1$, $\eta \sim -1$ in the analysis of the GZK threshold, one should sometimes confidently dismiss the relevant "conspiracy hypothesis", which (as stressed in subsection IV.D) would require a truly implausible role of a background of multi-TeV photons.

The adoption of commonly agreed criteria on how to handle these experimental bounds valid "up to conspiracies" would be an important asset for the new phase of quantum-gravity phenomenology which I am proposing.

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